

Session 8: Hypothesis Test II

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<http://www.nemoursresearch.org/open/StatClass/February2013/>

Chi-squared Test

Chi-square is a nonparametric statistical test commonly used to compare observed data with data we would expect to obtain according to a specific hypothesis

Good for:

- goodness of fit (purpose)

- association / independence (purpose)

- COUNTS and PROPORTIONS (data type)

Background

Table1	New drug A: low-dose	New drug A: high-dose	Control 1: music therapy_Bach	Control 2: no care or treatment at all
faint	20	10	40	50
no faint	15	45	30	15

Table2	DrPhil Drug1	Control: no care or treatment at all
faint	88	42
no faint	20	66

Your lab developed New drug A to improve patient outcome of surgery S. You reviewed the relevant lit, were inspired by a study by Dr. Phil et al, and decided to **define** "improve" by incidence of fainting and **measure** it by counting fainted participants in each of your study groups. You cross-tabulated the number of subjects by treatment groups in the tables.

Questions could be asked from your data

1. Does New drug A work?
2. How well does new drug A work compared to Dr. Phil's drug?

BOTH can be tested by the Chi-square test,
but HYPOTHESES are different

Test logic and null hypotheses

For Question 1:

Identify: chi-square test of association

Data: all your study groups

GLOBAL Ho: % fainted subjects is NOT DIFFERENT btw ALL study groups

For Question 2:

Task: need to establish basis of comparison between your and Dr. Phil's control groups: test demographic variables AND outcome (% faint)

WANT: large p-values showing no sig diff btw your & Dr. Phil's ctrl grps

Identify: chi-square test of goodness of fit

Data: your & Dr. Phil's ctrl grps

Ho: % fainted subjects is NOT DIFFERENT btw the 2 ctrl groups

Alternative hypotheses-ALWAYS 2-sided

Question 1: GLOBAL Ho: % fainted NOT DIFFERENT btw ALL study groups at 95% significance level, Ho will be rejected IF under the assumption that Ho is true, the probability of obtaining the data showing the proportions as radical as they are (definition of p-value) IS LESS THAN 5% for any of the following situations:

ctrl1 & 2 diff, high & low doses diff, high & ctrl1 diff, low & ctrl1 diff, high & ctrl2 diff, low & ctrl2 diff...

Mathematically express Ho:

% fainted ctrl1 = % fainted ctrl2 = % fainted high dose = % fainted low dose

Testing Procedure

	New drug A: low-dose	New drug A: high-dose	Control 1: music therapy_Bach	Control 2: no care or treatment at all	DrPhil Drug1	HistCtrl: no care or treatment at all
faint	20	10	40	50	88	42
no faint	15	45	30	15	20	66

	DrPhil Drug1	Control: no care or treatment at all
faint	88	42
no faint	20	66

Merge and answer both questions ONLY if justifiable, else focus on answering question 1.

Test app:

<http://www.quantpsy.org/chisq/chisq.htm>

Chi-square test requirements

1. Each observation is independent of all the others (i.e., one observation per subject);
2. "No more than 20% of the expected counts are less than 5 and all individual expected counts are 1 or greater" (Yates, Moore & McCabe, 1999, p. 734).

What to do when the requirements are not met

Req recap: #1 independent obs #2 at least 5 counts per cell

If #1 is not met, #2 does not matter. Set up a consultation session with a statistician.

If #1 is met but #2 is not met AND you only have a 2-by-2 table, go for Fisher's exact test.

If #1 is met but #2 is not met AND you have m-by-n groups, then merge groups OR analyze only the chi-square-test-eligible groups with the appropriate hypotheses, and proceed to the chi-square test

***NOTE: chi-square test does not work well when any proportions is close to 0 or 100%**

1	450
999	200

Example: Merging Groups

Table1	New drug A: low-dose	New drug A: high-dose	Control 1: music therapy_Bach	Control 2: no care or treatment at all
faint	2	10	40	50
no faint	15	45	30	15

CANNOT merge treatment groups and chi-square test if ask question about dosage-dependent effect of New drug A, but CAN merge if only ask question whether New drug A is effective compare to controls

<i>Table1 MERGED</i>	<i>New drug A</i>	<i>Control 1</i>	<i>Control 2</i>
<i>faint</i>	12	40	50
<i>no faint</i>	60	30	15

Fisher's Exact Test for 2-by-2 table

Test based on the ideology of permutation

Good for:

association / independence (purpose)

counts (proportions), at least $n=1$ per cell (data type)

<http://www.quantitativeskills.com/sisa/statistics/fisher.htm>

The p-value from the test is computed as if all column- and row-totals are fixed

*If run Fisher's exact test and chi-square test on the same 2-by-2 table, Fisher's 2-sided p-value > chi-square p-value

* Freeman-Halton extension of the Fisher exact test could be applied to m-by-n tables. (Consult a statistician)

<http://vassarstats.net/fisher3x3.html>; <http://vassarstats.net/fisher2x4.html>; <http://vassarstats.net/fisher2x3.html>

Odds, Odds Ratios (OR): From Frequency to Probability (Pr)

Frequency: With no assumptions about the coin's fairness, toss it 10 times, heads show up in 3 tosses. Odds of getting a head is $3/(10-3)=3/7$

Translate frequency into probability:

3 -> $3/10 = 30\%$; 7 -> $7/10 = 70\%$; 10 = 100%

Calculate odds using probability: $0.3/0.7=3/7$

Suppose I have another coin, out of 10 tosses 5 heads show up. Odds of getting head from coin2=
?

Between odds, OR, Pr and frequency

Can calculate Pr, odds and OR from frequency

Can calculate odds and OR if know Pr

CANNOT calculate frequency (actual data points) from only knowing Pr, odds or OR (summary statistics).

CAN calculate frequency from only knowing Pr, odds or OR IF KNOW RELEVANT SAMPLE SIZE.

Example: if only know coin 1 Pr of head = 0.3, cannot know # heads actually being tossed; If know coin 1 Pr of head = 0.3 AND there were a total of 10 tosses, can know # heads being tossed; If know coin 1 Pr of head = 0.3 AND 3 heads were tossed, can know total # coin tosses.

Example

low-dose % faint=11.7%

high-dose % faint=18.2%

$$OR = (2 \times 45) / (15 \times 10) = 0.6$$

Interpretation: odds of fainting of those on New drug A low-dose is 0.6 times (60% of) the odds of fainting of those on New drug A high-dose.

$$OR = (15 \times 10) / (2 \times 45) \text{ approximately } 1.7$$

Interpretation: odds of fainting of those on New drug A high-dose is 1.7 times the odds of fainting of those on New drug A low-dose.

Table1	New drug A: low-dose	New drug A: high-dose
faint	2	10
no faint	15	45

****Realize that
1/0.6=1.7 and
1/1.7=0.6**

Interpreting OR

- OR is a ratio.
- $OR(\text{rain today} / \text{rain tomorrow})=1$ means the odds that it will rain today and the odds that it will rain tomorrow are equal (both days could have 10%, 79%, etc chances of raining)
- $OR(\text{rain today} / \text{rain tomorrow})=3$ means the odds that it will rain today is 3 times the odds it will rain tomorrow
- $OR(\text{rain today} / \text{rain tomorrow})=0$ means the odds today will rain is either 0 or 100%
- If $OR(\text{rain today} / \text{rain tomorrow})=3$,
 $OR(\text{rain tomorrow} / \text{rain today})=1/3$;
- If $OR(\text{rain today} / \text{rain tomorrow})=0.2$,
 $OR(\text{rain tomorrow} / \text{rain today})=1/0.2=5$

Testing H_0 in terms of OR

In terms of differences:

H_0 states $\text{diff}(\text{group A vs B})=0$;

In terms of ratios:

H_0 states $\text{ratio}(\text{group A vs B})=1$;

OR calculation link: <http://statpages.org/ctab2x2.html>