Session 7: Hypothesis Test, Part I

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Session 7 Flow

1. Terminologies

A. Null and alternative hypotheses

B. One and two-sided tests

C. Type I and II errors, significance level, power

C. Rejection region and confidence interval

E. P-value, test statistic

F. Degree of freedom

2. 1- and 2-sample paired and unpaired t tests; Wilcoxon (signed-rank) test; Mann Whitney test

Null and alternative hypotheses

Null (Ho): hypothesis to be tested

Alternative (Ha): complementary to Ho, so that the **mutually exclusive** pair (Ho and Ha) together **exhaust ALL** possible outcomes.

Typical Ho: No association between X and Y, outcome is not different in placebo and treatment groups, proportion of those who have symptoms is not different among the exposed and non-exposed, etc.

Null and alternative hypotheses-EXAMPLE

Null: outcome is not different in placebo and treatment groups

One-sided: treatment group's outcome BETTER than placebo's

One-sided: treatment group's outcome WORSE than placebo's

Two-sided: treatment group's outcome DIFFERENT FROM placebo's

One and Two-sided tests- QUANTIFIED

One-sided: treatment group's thromboxane level lower than placebo's

One-sided: treatment group's thromboxane level higher than placebo's

Two-sided: treatment group's thromboxane level lower or higher than placebo's

Why running two-sided tests is standard practice

p-value from a two-sided test is 2 times the p-value from a one-sided test -> guard against type I error (claiming stat sig difference when there is none)

Ho in 2 groups vs GLOBAL Ho in >2 groups

Suppose 50% subjects in treatment group are on high-dose scheme, 50% are on low-dose

Questions could be asked:

1. Does the drug has ANY effect?
2. Is there any evidence for dose-dependent effect?

Pop quiz: formulate Ho and Ha for question 1

GLOBAL Null Hypothesis

When comparing more than 2 groups, has more than 3 possible results/inferences

Example: placebo, treatment-low-dose, treatment-high-dose

Possible results: Let's enlist all 9

GLOBAL Null Hypothesis

Placebo **L**ow-dose **H**igh-dose

P=L=H	P=L<H	P=L>H
P<L=H	P<L<H	P<L>H
P>L=H	P>L<H	P>L>H

Shortcut:

possible results = 3 to the power of (# grps - 1)

5 groups=3^4=3x3x3x=81 possible results when comparing groups

GLOBAL Null Hypothesis

In ANOVA, ANCOVA, etc.

Global Ho: NONE of the groups are different

Could be rejected if

P=L=H	P=L<H	P=L>H
P<L=H	P<L<H	P<L>H
P>L=H	P>L<H	P>L>H

Application to Homework Data Set

Case 1 Theory: Prevalence of family history of diabetes is associated with BMI

Case 2 Theory: Prevalence of family history of diabetes is differentially associated with BMI in different racial/ethnic groups (suppose C, A, AA, O 4 racial/ethnic groups)

Case 1 Ho:

Case 1 Ha(s):

Case 2 Ho:

Case 2 Ha(s):

Type I and II Errors

Type 1 error: finding stat sig diff when there is none (rejecting null when null is true)

Type 2 error: not finding stat sig diff when there is (accepting null when null is false)

Significance level = PROBABILITY of making type 1 error. "a=0.05" is a 5% PROBABILITY

Power = PROBABILITY of rejecting null when null is false = 1 - Prob(type 2 error)

P-value and Test Statistic

- P-value: PROBABILITY of obtaining a test statistic at least as extreme as the one that was actually observed, assuming (not GIVEN) Ho is true.
- Test statistic: a summary statistic of a set of data that reduces the data to one or a small number of values that can be used to perform a hypothesis test.
- t-test statistic

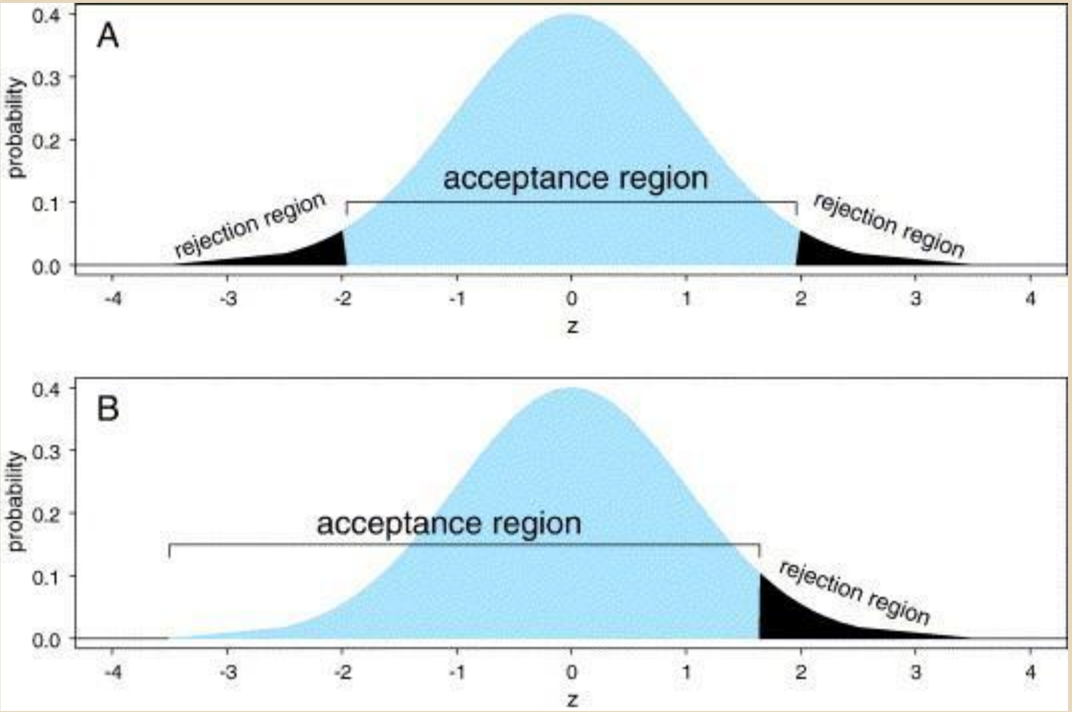
$$t = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}$$

Rejection Region and Confidence Interval

Test statistic: acceptance region + rejection region

- Critical value: value of test statistic that leads to rejection of Ho

Sample: confidence interval + elsewhere



Degree of Freedom (DF)

In most cases, larger sample size <-> greater DF

Why Important--Example: t test

At a=0.05 with 2 degrees of freedom (n=3), to reject Ho, need t test statistic > 2.920

At a=0.05 with 20 degrees of freedom (n=21), to reject Ho, need t test statistic > 1.725

NEED A VERY LARGE NUMERATOR, OR A VERY SMALL DENOMINATOR, OR BOTH

T test df   t test online

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

t test: 1- vs 2-sample

In a 1-sample t test, the entire data set is treated as 1 group, *mean*, *stdev* and *n* are calculated from the entire data set and compare to a KNOWN μ_0 (i.e. Value of μ_0 NOT from data)

A 2-sample t test is either paired or unpaired

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

1-sample t test Ho: the MEAN of data equals μ_0

2-sample t test Ho: the MEANS of the 2 groups are equal

t test: paired vs unpaired 2-sample

In a 2-sample paired t test, test statistic is calculated from the DIFFERENCE between paired observations from each group and # subjects from the 2 groups

$$t = \frac{\Sigma d}{\sqrt{\frac{n\left(\Sigma d^2\right) - \left(\Sigma d\right)^2}{n-1}}}$$

(d=diff btw paired obs)

In a 2-sample unpaired t test, test statistic is calculated from mean stdev n of each of the 2 groups

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Paired vs unpaired t test

Quick check for paired vs unpaired:

If re-assigning ID number from one of the groups is okay, then unpaired. If not, paired.

TYPICAL PAIRED data: baseline-endline (not okay to re-assign baseline ID), matched case-control (cannot break the matching), etc.

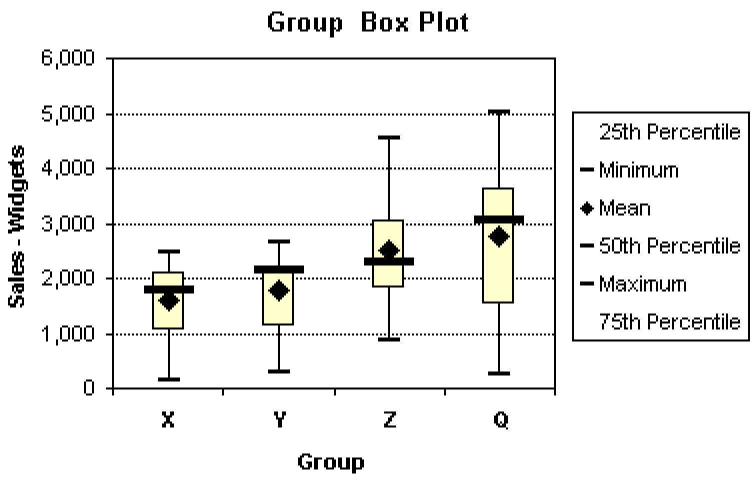
t test assumptions

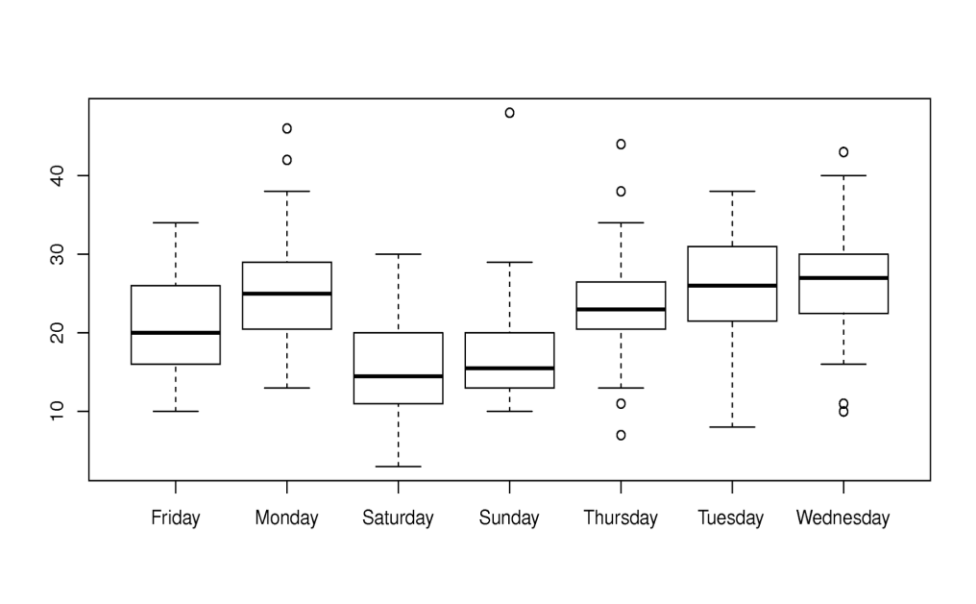
1. No group A subject is in group B
2. Dependent variable is continuous
3. Each observation (or pair of observations) of the dependent variable is independent of the other observations of the dependent variable.
4. Random sampling
5. Dependent variable follows a normal distribution (1-sample & 2-sample unpaired); Dependent variable follows normal distribution with EQUAL VARIANCE in 2-sample paired t test

Assessing assumptions: When Box Plot Comes in Handy

Location of means?

Difference in variances (stdev) ?





Formal Tests for Equality of Variance and Normal Distribution

F test: Ho: variances in 2 groups are equal

Shapiro-Wilk test: Ho: data is normally distributed

If T tests assumptions are violated...

Parametric test	Non-Parametric analogue
1-sample t test	NONE
Paired 2-sample t test	Wilcoxon (signed-rank) test
Unpaired 2-sample t test	Mann Whitney (U) test

Wilcoxon (signed-rank) test

Ho: MEDIAN diff between pairs in each group=0

Mann Whitney test (Wilcoxon rank-sum test)

Ho: Median diff between GROUPS=0

Ordinal (Ranked) dependent variables

Wilcoxon test

Ho: MEDIAN diff **in rank** between pairs in each group=0

Mann Whitney test

Ho: Median diff **in rank** between GROUPS=0

Parametric vs Non-Parametric Tests

Non-parametric tests

Pro: robust to outlier, less assumptions

Con: larger p-value compare to parametric analogues, does not work well when n is small, less sensitive and efficient if data meets assumptions of parametric analogues

Hypothesis Test Decision Tree (2 groups, continuous dependent variable)

1. determine variable type->2. calculate appropriate descriptive statistics->3. generate appropriate plots->4. assess parametric test assumptions

   ->5a. if assumptions are met, then parametric

   ->5b. if assumptions are violated, check descriptive statistics in each group, examine extreme values, remove outliers when appropriate, re-start at step 2